



## Chapter 10: Historical Topics

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# Chapter 10: Topics of Historical Interest, Heuristic Arguments & Common Misconceptions.

## 10.1 The Contributions of Hoerl and Kennard.

10.1.1 Characterization of the “Ordinary” Ridge Shrinkage Path.

10.1.2 The Hoerl-Kennard “too-longness” argument.

10.1.3 The Hoerl-Kennard “existence theorem.”

10.1.4 Heuristic “Fixed-Point” Arguments.

## 10.2 The distinction between “weak” data and “ugly” data.

## 10.3 The “Chain-Rule” Argument of Obenchain-Vinod.

My joint paper with Rick Vinod [Obenchain and Vinod(1974)] was my first work devoted exclusively to shrinkage regression. Unfortunately, this paper turned out to be highly “controversial” in several senses. For example, we had to revise it twice just to get it released for publication from Bell Telephone Laboratories, then we performed another major revision between submissions to two different journals. But it was never accepted for publication!

As is clear from the style and grammar of these early manuscripts, I (rather than Rick) did most of the writing, but our original intent was simply to explain/defend a concept due to Rick [Vinod(1971)] called the **Chain-Rule Argument**. Much of the notation and terminology of these manuscripts was new/innovative at that time; many of the fundamental relationships described in Chapters 2 and 3 of this book first appeared in these manuscripts. Furthermore, the concepts of PCSA and of plotting TRACES versus MCAL were first introduced there. Ultimately, just about everything from these papers, **except** the chain-rule argument, got published in Obenchain(1975,1976) and/or in Vinod(1976).

shrunk coefficients as constrained partial derivatives.

Cox(1971) argues that a reparameterization of Scheffe coefficients for mixture experiments is desirable because it allows the new coefficients to be interpreted as directional derivatives. Of course, Cox is almost surely assuming that the data to be fitted come from a "designed experiment."

#### **10.4 "Fictitious Data Augmentation" Arguments.**

#### **10.5 Preliminary-Test Estimation.**

Furthermore, my references to "preliminary testing" came, politically speaking, at a most inappropriate time. The highly influential work of Bock, Yancey, and Judge(1973) was being widely interpreted, around then, as sounding a "death nell" for preliminary test methodology!

In my paper, I had adopted what a 1950's television critic described as the "BEVERLY HILLBILLY'S STRATEGY," which was to... "Aim Low, and Hit Your Mark." I had devised a statistically valid testing procedure for what McDonald suggested I call the UNIFORM SHRINKAGE HYPOTHESIS, under which least squares regression coefficients have mean-squared-error optimal relative magnitudes. This test (and its associated approximate F-distribution) provided ample power to detect ill-conditioned regression problems and, thereby, "reject" least-squares coefficients on the grounds that their relative magnitudes might be potentially misleading. Apparently, my testing procedure also provides almost unlimited potential for ABUSE.

#### **10.6 Latent-Root Regression Methods.**

#### **10.7 Adjusting Correlations Among Regression Coefficients.**

Of all the papers I ever "volunteered" to review for internal clearance by Bell Telephone Laboratories, Tukey(1975) provides my most vivid memories of the strengths and weaknesses of a "peer-review" process. I suspect that John Tukey was least happy when I described the methodology of his Section 6 as a "new form of ridge regression."

#### **10.8 Best Linear and Quadratic Estimators of One.**

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